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LEAST SQUARES PREDICTION FOR MIXED AUTOREGRESSIVE MOVING AVERAG--ETC(U)
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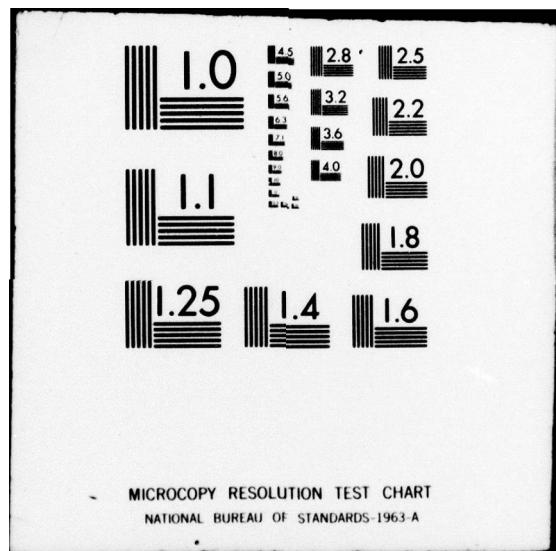
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TEST SQUARING PREDICTION FOR MIXED AUTOREGRESSIVE

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Two A & M Research Foundation
Project No. 3638

"Multiple Time Series Modeling and Time Series Theoretic Statistical Methods"

Professor Franklin Parsons, Principal Investigator

Answers for public failure: disaster preparedness

LEAST SQUARES PREDICTION FOR MIXED AUTOREGRESSIVE

MOVING AVERAGE TIME SERIES

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Keywords: Mixed Time Series, Toeplitz Matrix, Modified Cholesky Decomposition

LANGUAGE

ISO Fortran

DESCRIPTION AND PURPOSE

Let $y(1), \dots, y(T)$ be a sample realization of a mixed autoregressive moving average process $(y(t), t = 0, \pm 1, \dots)$ of order (p, q) , i.e., $y(t)$ satisfies

$$\sum_{j=0}^p a(j)y(t-j) = \sum_{k=0}^q b(k)\epsilon(t-k), \quad t = 0, \pm 1, \dots$$

for constants $p, q, a(0) = b(0) = 1, a(1), \dots, a(p), b(1), \dots, b(q)$, where $\epsilon(\cdot)$ is a white noise time series of zero mean, uncorrelated random variables with variance σ^2 . The zeros of the complex polynomials $s(z) = \sum_{j=0}^p a(j)z^j$ and $b(z) = \sum_{k=0}^q b(k)z^k$ are assumed outside the unit circle.

Subroutine **MSPD** calculates least squares predictors $y(t+v|t)$ of $y(t+v)$ given $y(1), \dots, y(t)$ for

$$v = v_p, \dots, v_L \quad \text{for } t = t_p, \dots, t_L. \quad (1)$$

NUMERICAL METHOD

The algorithm given by Pagano and Parzen (1973) (based on the work of Whittle (1963)) is used. Let $X(t) = \sum_{j=0}^p a(j)y(t-j)$, $t = p+1, \dots, T$.

Then the $X(\cdot)$ are a sample realization of a pure moving average process of order q and $\tilde{X}^T = (X(p+1), \dots, X(T))$ has the symmetric Toeplitz correlation matrix Γ where

$$\Gamma_{jk} = \begin{cases} 1 & j = k \\ \rho(|j - k|) & |j - k| = 1, \dots, q \\ 0 & |j - k| > q \end{cases}$$

and $\rho(v) = \sum_{k=0}^q b(k)s(k + |v|) / \sum_{k=0}^q b^2(k) = R(|v|)/R(0)$, $|v| \leq q$.

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Let $\Gamma = L\Gamma^T$ be the modified Cholesky decomposition (Whittle (1967)) of Γ , i.e., L is a unit lower triangular $(T-p) \times (T-p)$ matrix (L_{ij}) and D is a $(T-p) \times (T-p)$ diagonal matrix (D_{ij}) . Define $\rho^T = (\rho(p+1), \dots, \rho(T))$ by $\rho^T = X(t) - \frac{1}{T-p} \sum_{t=p+1}^T X(t-p-k) \epsilon(t-k)$, $t = -q+1, \dots, T$, i.e., $\rho^T = \tilde{X}$.

Then $y(t+v|t) = X(t+v|t) - \frac{1}{T-p} \sum_{j=1}^p a(j)y(t+v-j|t)$ where

$$X(t+v|t) = \begin{cases} \sum_{k=0}^q b(k)\epsilon(t+v-k), & v = 1, \dots, q \\ 0, & v > q \end{cases}$$

and $y(v|t) = y(t)$ if $v \leq 0$.

Thus the algorithm essentially consists of finding successive rows of the matrix L , using as little storage space as possible. This is done by noting:

1) the k th row of L has $\eta_1 = \max(k-q-1, 0)$ leading zero elements followed by $\eta_2 = \min(k-1, q)$ elements, followed by a 1, followed by $\eta_3 = \eta_1 - \eta_2 - 1$ zeros. Thus only η_2 elements need be stored for any row.

2) to find the k th row of L , at most the previous q rows are necessary.

3) Bauer (1955) has shown that as k gets large, $L_{k,k-j} \rightarrow R(j)$ and $R_{kk} \rightarrow \sigma^2 R(0)$. Thus for a fixed δ , there should be an integer N such that

$$|L_{k,k-j} - R(j)| < \delta \\ |R_{kk} - \sigma^2 R(0)| < \delta \quad (2)$$

for $j = 1, \dots, q$, $k \geq N$. Thus only the first N rows of L need be computed.

MSPD calls auxiliary subroutine **MORN** to find $\rho(p+1), \dots, \rho(T)$ using only a $(q+1) \times (q+1)$ work space for computing L . **MORN** is designed so that it can be used also for performing Bauer's algorithm or to find the modified Cholesky decomposition of the symmetric Toeplitz matrix whose first row is $(1, \rho(1), \dots, \rho(q))$.

MSPD is designed to perform prediction for pure autoregressive time series if $q = 0$ and for pure moving average time series if $p = 0$.

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STRUCTURE	
SUBROUTINE MARSH (IOPTE, IOP, NQ, DEL, ITERS, IROWS1, Y, BETA, SIGSQ, IROFS2,	
NQ, NN, IROWS1, IROWS2, AL, D, NEL, R, N, E, TPD, IPAUT)	
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STRUCTURE

SUBROUTINE MARSH (IOPTE, IOP, NQ, DEL, ITERS, IROWS1, Y, BETA, SIGSQ, IROFS2, NQ, NN, IROWS1, IROWS2, AL, D, NEL, R, N, E, TPD, IPAUT)

Auxiliary algorithm

SUBROUTINE MARSH (IOPTE, IOP, NQ, DEL, ITERS, IROWS1, Y, BETA, SIGSQ, IROFS2, R, N, NEL, D, AL, E, IPAUT)

Formal parameters

Formal parameters

IOPTE	Integer	input: option switch equal to: 1 if coefficients are to be calculated 0 if coefficients are inputted
IOP	Integer	input: option switch equal to: 1 if c's are to be calculated 0 if e's not to be calculated
NQ	Integer	input: order of moving average convergence criterion: see (2)
NN	Real	input: order of correlation matrix dimension of Y and E arrays, give by: > 1 if IOPTE = 0 > ITERS if IOPTE = 1
IROWS1	Integer	input: (If IOPTE = 1) or workspace (if IOPTE = 0): data array
IROWS2	Integer	input: (If IOPTE = 0) or output (if IOPTE = 1): moving average coefficients
Y	Real Array (IROWS1)	input: (If IOPTE = 0) or output (if IOPTE = 1): variance of c(*) row dimension of AL and dimension of D in DIMENSION statement of calling program
MMAT	Real Array (NQ)	input: (If IOPTE = 1) or output (if IOPTE = 0): moving average auto- covariances for lags 1, ..., NQ corresponding to BETA, SIGSQ
SIGSQ	Real	input: (If IOPTE = 1) or output (if IOPTE = 0): moving average vari- ance corresponding to BETA, SIGSQ
IROWS2	Integer (> NQ)	input: (If IOPTE = 1) or output (if IOPTE = 0): workspace
R	Real Array (NQ)	input: (If IOPTE = 1) or output (if IOPTE = 0): workspace
AL	Real Array (NQPL, NQPL)	output: workspace
D	Real Array (NQPL)	output: workspace
AL	Real Array (NQPL)	output: workspace
R	Real Array (NQPL)	output: workspace
N	Real	output: moving average variance corre- sponding to MMAT, SIGSQ
Y	Real Array (IROWS2)	output: predictors stored as $y(t_p + v_1 t_p), \dots$
X	Real Array (IROWS2)	$y(t_p + v_1 t_p), \dots$
X	Real Array (IROWS2)	$y(t_p + v_1 t_p), \dots$
TPD	Real Array (NQ)	$y(t_p + v_1 t_p)$
IPAUT	Integer	output: fault indicator, equal to: 0 if proper execution 1 if NPL < 1 2 if NPL < 1 3 if NN < (NPL - NTF + 1) (NPL - NTF + 1) 4 if IROWS1 < NPL 5 if correlation matrix singular

ROUTINE MACV1 (NQ, BETA, SIGSQ, R, RQ, ITABLE)
 Description: MACV1 calculates moving average autocovariances corresponding to moving average parameters.

Formal parameters

SQ	Integer	Input: order of moving average
BETA	Real Array (NQ)	Input: moving average coefficients
SIGSQ	Real Array (NQ)	Input: variance of $c(t)$
R	Real Array (NQ)	Output: autocovariances for lags 1, ..., NQ
NQ	Real	Output: variance of moving average
ITABLE	Integer	Faulty indicator, equal to: 0 if proper execution 1 if NQ < 1

RESTRICTIONS, TIME, NECESSARY STORAGE

If the zeros of $h(z)$ are not outside the unit circle, the correlation matrix Γ is not positive definite. This is manifested by a diagonal element of its modified Cholesky decomposition becoming nonpositive. Subroutine MACV1 tests for this by checking for diagonal elements being $< EPS$ (specified in data statement). If one is found, ITABLE is set to 5 and executive stops.

Subroutine MACV1 requires $(p+1) + (q+4)(q+1) + 37 + 3N$ storage locations. The number of operations is approximately $(t_L - p)(p + q) + Nq(q + 1)/2 + (t_L - t_q + 1)(v_L P + q + (q - 1) + \dots + \max(1, q - v_L + 1))$, where N is the number of rows of L before convergence. N increases as the smallest zero of $h(z)$ approaches the unit circle.

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Outline of Algorithm for Subroutine MAORTH

Subroutine MAORTH is used to find a vector $\epsilon = (c(1), \dots, c(T))^T$, by $Y_g = Y$, where $Y^T = (Y(1), \dots, Y(T))$ is a sample realization of a moving average process of order q with coefficients $\beta(1), \dots, \beta(q)$ and noise variance σ^2 , and L is the $(T \times T)$ unit lower triangular matrix of the modified Cholesky decomposition $\Gamma_{q,T} = LDL^T$ of the $T \times T$ symmetric band Toeplitz correlation matrix $\Gamma_{q,T}$ of Y . Thus the (j, k) th element of $\Gamma_{q,T}$ is given by

$$(\Gamma_{q,T})_{jk} = \begin{cases} 1 & \text{if } j = k \\ \rho(|j - k|) & |j - k| = 1, \dots, q \\ 0 & |j - k| > q \end{cases}$$

where $\rho(v) = R(v)/R(0)$ and

$$R(v) = \sigma^2 \sum_{k=0}^{q-v} R(k)R(k+v), \quad v = 0, \dots, q$$

(Subroutine MACV1 calculates $R(0), R(1), \dots, R(q)$ given $q, \sigma^2, \beta(1), \dots, \beta(q)$)

Subroutine MAORTH can also be used to calculate $\sigma^2, \beta(1), \dots, \beta(q)$ if $R(0), R(1), \dots, R(q)$ are inputted. Thus if ITOPR = 1, the β 's are input while if ITOPR = 0, the β 's and σ^2 are input.

Further, one need not calculate the c 's if the subroutine is used only for Bauer's algorithm (ITOPR = 1). Thus if ITOPR = 1, the c 's are calculated (and the Y 's are input), while if ITOPR = 0, the c 's are not calculated (and the Y 's need not be inputted). The dimension of E and Y (the mnemonic for c, Y) is IRONSI which can be 1 if ITOPR = 0.

Finally, subroutine MORTH can be used to merely find the modified

Cholesky decomposition of $\Gamma_{q,T}$ by inputting $\Gamma_{q,T}$, $R(0)$, $R(1)$, ..., $R(q)$, $0, \dots, 0$, letting $NQ = T - 1$, $INFO = T$, $INFO = 1$, and $INFO = 0$.

Let $A = (a_{jk})$ be a real symmetric ($T \times T$) matrix and $L = (l_{jk})$.

$D = \text{diag}(d_1, \dots, d_T)$ be the factors in the Modified Cholesky decomposition of A , i.e., $A = LD^T$. Then

$$l_{11} = 1, \quad d_1 = a_{11} \\ l_{21} = \frac{r_{21} - \sum_{j=1}^{q-1} l_{1j} l_{1j}^T}{d_1}, \quad 1 < 1 = 2, \dots, T \\ l_{31} = \frac{r_{31} - \sum_{j=1}^{q-1} l_{1j} l_{1j}^T - \sum_{j=2}^{q-1} l_{2j} l_{2j}^T}{d_1}, \quad 1 < 1 = 3, \dots, T \\ \vdots \\ l_{k1} = \frac{r_{k1} - \sum_{j=1}^{q-1} l_{1j} l_{1j}^T - \sum_{j=2}^{q-1} l_{2j} l_{2j}^T - \dots - \sum_{j=k-1}^{q-1} l_{(k-1)j} l_{(k-1)j}^T}{d_1}, \quad 1 < 1 = k, \dots, T$$

For $A = \Gamma_{q,T}$, $l_{11} = 0$ if $k - 1 > q$; in fact the nonzero elements of row k of L are

$$l_{k1}, l_{k2}, \dots, l_{kq_1}, l_{kq_1+1}, l_{kq_2}, \dots, l_{kq_2+1}$$

where $q_1 = \min(k - q - 1, 0)$, $q_2 = \min(k - 1, q)$.

The equations given above become

$$l_{kq_1+1} = \frac{r_{kq_1+1} - \sum_{j=1}^{q_1} d_{1j} l_{1q_1+1} - \sum_{j=2}^{q_1} d_{2j} l_{2q_1+1} - \dots - \sum_{j=q_1}^{q_1} d_{q_1j} l_{q_1q_1+1}}{d_{q_1+1}}, \quad 1 = 1, \dots, q_1$$

$$d_k = 1 - \sum_{j=1}^{q_1} d_{1j}^2 l_{1q_1+1}^2 - \sum_{j=2}^{q_1} d_{2j}^2 l_{2q_1+1}^2 - \dots - \sum_{j=q_1}^{q_1} d_{q_1j}^2 l_{q_1q_1+1}^2$$

From these equations it is possible to note:

$$l_{kq_1+1} = \frac{r_{kq_1+1} - \sum_{j=1}^{q_1} d_{1j} l_{1q_1+1}}{d_{q_1+1}}$$

Thus in calculating row k of L and d_k , one needs only rows

$q_1 + 2, \dots, q_1 + q_2 - k - 1$ of L and $d_{q_1+1}, \dots, d_{q_1+q_2}$, i.e., at most the previous q rows of L and the previous q elements of D . However, $q + 1$ rows are stored so that the decomposition of $\Gamma_{q,q+1}$ can be obtained.

Let $NQ = q$, $NQ1 = q + 1$. Subroutine MORTH uses the constant DK and the arrays $D(NQ1)$, $W(L(NQ1))$, $AL(NQ1)$, $NQ1$) to determine the rows of L and the diagonal elements of D :

Step 1 : Initialize $D(1) = 1$, $AL(1, 1) = 1$, $I = 1$, $NQ1$
Calculate $R(1) = R(1)/NQ$, i.e. the autocorrelations

Step K :

Calculate $l_{k,q_1+1}, \dots, l_{k,q_1+q_2}, d_k$ and store in $W(L(1), \dots, W(L(NQ))$.

DK

If $K \leq NQ1$: put $W(L)$ into K^{th} row of AL , DK into $D(K)$, and go to next row

If $K > NQ1$: shift down by 1 the 2^{nd} through $NQ1^{\text{th}}$ rows of AL and 2^{nd} through $NQ1^{\text{th}}$ elements of D . Put $W(L)$ into $NQ1^{\text{th}}$ row of AL and DK into $D(NQ1)$.

It remains to determine where the j^{th} element of row I of L is located at step K. The elements of the I^{th} row of L that aren't identically zero or one are $L_{I,M1}, \dots, L_{I,L-1}$, where $M1 = \max(I - M0, 1)$. They are stored in the first through $\min(I - 1, M0)$ th elements of row $(I - M0)$ of AL at step K, i.e., $L_{I,M0+1}$ is in $AL(I - M0, M)$ or $L_{I,I}$ is in $AL(I - M0, J - M0 + 1)$ at step K.

Thus the equations given above can be written:

For step $K \geq 2$: Let $M1 = n_1 = \max(K - M0 - 1, 0)$, $M2 = n_2 = \min(K - 1, M0)$, $M3 = \max(K - M0, 1)$, $M4 = \max(K - M0 - 2, 0)$.

$$AL(I, I) = \frac{a(I-n_1-1)}{a_{n_1+1}} = \frac{a(I-M1-1)}{a_{M1+1-M0}}$$

If $M3 < 2$, go to 4.

$$a(I-M1-J) = \frac{a_{n_1+1}^J a_{n_1+1}^T a_{n_1+1}^T a_{n_1+J}}{a_{n_1+J}}$$

$$AL(I, I) = \frac{a_{n_1+1}^{J-1} a(I-n_1-J) - a_{n_1+1}^T a_{n_1+1}^T a_{n_1+J}}{a_{n_1+J}^T a_{n_1+J}}$$

$$a(I-M1-J) = \frac{a(I-n_1-J) - a(I-n_1-M1+1) a_{M1+1-M0}^T a_{M1+1-M0} (I-n_1-J)}{a_{M1+1-M0}^T a_{M1+1-M0}}$$

$$a_{M1+1-M0} = 1 - \sum_{j=1}^{M1} a_{n_1+J}^T a_{n_1+J}^T a_{n_1+J}$$

$$= 1 - \sum_{j=1}^{M3} a_{(M1+J-M0)}^T a_{(M1+J-M0)} (J) a_{M1+1-M0}^T a_{M1+1-M0}$$

Summary of MARSH (let K represent c)

Initialise:
 $\frac{M0}{M1} = 1.1 - 10$
 If $10PTK = 0$, calculate $M0$, $R(1), \dots, R(M0)$ via MACY1.
 Store $a(1), \dots, a(M0)$ in $R(1), \dots, R(M0)$
 $D(1) = 1$
 $AL(I, I) = 1$, $I = 1$, $M0$
 If $10PTK = 1$, $E(1) = Y(1)$

Do 130 K = 2, ITMAX
 Calculate $M0$, IK
 If $10PTK = 1$, calculate $E(K)$
 If $IK < M0$, E , q , r not positive definite. Go to 199
 $K \leq M0$: put $M0$, IK into AL , D . Go to 130
 $K > M0$: do shifting
 $10PTK = 1$: check convergence of rows of AL .
 Yes : Form $M0$, $SIGSQ$. Go to 150
 No : Go to 130
 $10PTK = 0$: check convergence of AL to $BETA$
 Yes : Go to 150
 No : Go to 130

130 CONTINUE

- If Loop is performed for all K, then the algorithm is done except if $10PTK = 1$ in which case $BETA$, $SIGSQ$ are assigned values from last row of AL and D and Go to 199

150 CONTINUE

- If $10PTK = 1$, calculate the rest of the E 's using $BETA$

199 Reform autocovariances
RETURN

Outline of Algorithm for Subroutine MRD

Let $Y(1), \dots, Y(T)$ be a sample realization of length T of a mixed autoregressive moving average time series of order (p, q) with autoregressive parameters $\alpha(1), \dots, \alpha(p)$ and moving average parameters $\theta(1), \dots, \theta(q)$, and σ^2 .

Subroutine MRD calculates the least squares predictors

$$\begin{aligned} YPD(1) &= Y(t_p + v_p | t_p), \quad YPD(2) = Y(t_p + v_p + 1 | t_p), \dots, \quad YPD(M1) = Y(t_p + v_L | t_p), \\ YPD(M1 + 1) &= Y(t_p + 1 + v_p | t_p + 1), \quad YPD(M1 + 2) = Y(t_p + 1 + v_p + 1 | t_p + 1), \dots, \\ YPD(2*q1) &= Y(t_p + 1 + v_L | t_p + 1), \\ &\vdots \\ YPD((M2 - 1)*M1 + 1) &= Y(t_L + v_p | t_L), \dots, \quad YPD(MM) = Y(t_L + v_L | t_L), \end{aligned}$$

where $M1 = v_L - v_p + 1$, $M2 = t_L - t_p + 1$, $MM = M1 * M2$.

The predictor $Y(t + v | t)$ is given by

$$\begin{aligned} Y(t + v | t) &= \begin{cases} \sum_{j=1}^p \alpha(j)Y(t + v - j | t) & p \neq 0, q \neq 0 \\ \sum_{j=1}^p \alpha(j)Y(t + v - j | t) \\ \sum_{k=1}^q \theta(k)Y(t + v - k | t) & p \neq 0, q = 0 \\ 0 & p = 0, q \neq 0 \\ 0 & p = 0, q = 0 \end{cases} \end{aligned}$$

where:

(a) $Y(r|s) = Y(r)$

$$(b) \quad Y(t + v | t) = \begin{cases} \sum_{k=1}^q \theta(k) \epsilon(t + v - k) & v = 1, \dots, q \\ 0 & v > q \end{cases}$$

(c) $\epsilon = (\epsilon(p+1), \dots, \epsilon(T))^T = L_{q,T-p}^{-1} \xi$, where

$L_{q,T-p}$ is the unit lower triangular matrix in the modified Cholesky decomposition of the $(T-p) \times (T-p)$ correlation matrix $\Gamma_{q,T-p}$.

(d) $\xi = (X(p+1), \dots, X(T))^T$, where

$$X(t) = \sum_{j=0}^p \alpha(j)Y(t-j), \quad t = p+1, \dots, T.$$

Note that $NP1 = p+1$ and $NP2 = q+1$ are input rather than p and q so that the dimension of $ALPHA = \alpha$ and $BETA = \beta$ can't be zero without stopping execution.

If $q = 0$, the $X(t+v|t)$ (and thus the ϵ 's) are not needed.

The predictors $Y(t+v|t)$ satisfy a difference equation for fixed t .

Thus $Y(t+1|t), \dots, Y(t+v-1|t), X(t+1|t), \dots, X(t+v-1|t)$,

$Y(t|t) = Y(t), \dots, Y(t-p+1|t) = Y(t-p+1), \text{ and } \epsilon(t-q+1), \dots, \epsilon(t)$ are required to calculate $Y(t+v|t)$. If $t = q+1 \leq p+1$ or $t = p+1 < 1$ these values cannot be obtained. This is also true if $t > T$. Thus if $t < p+q$ or $t > T$ or $p+q = 0$, the predictors $Y(t+v|t), \dots, Y(t+v_L|t)$ are assigned the value zero.

Summary of MRD

If $NQ \neq 0$ calculate $E(NP+1), \dots, E(NQ)$ via NAGORT

Do 150 $NT = NTP, NTL$
 If $NT < NP + NQ$ or $NT > NP$ or $NT > T$ or $NP + NQ = 0$ set predictors equal to zero and go to 150

If $NP \neq 0$ put $Y(NT - NP + 1), \dots, Y(NT)$ into $X(1), \dots, X(NP)$.
 Calculate $Y(NT + 1|NT), \dots, Y(NT + NQ|NT)$ and store in $X(NP+1), \dots, X(NP+NQ)$.

Put $X(NP + NPF), \dots, X(NP + NVL)$ into $YPD((NT - NTF) + NL + 1)$,

$\dots, YPD((NT - NTF + 1) + NL)$, where $NL = NVL - NPF + 1$.

150 CONTINUE

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SUBROUTINE MXPD(NPP1,NQP1,ALPHA,BETA,SIGSQ,NOBS,Y,DEL,NTF,
1NVL,NVF,NVL,NN,IROWS1,IROWS2,AL,D,WKL,R,R0,E,X,YPD,IFault)

C THIS SUBROUTINE CALCULATES LEAST SQUARES PREDICTORS FOR A MIXED
C AUTOREGRESSIVE MOVING AVERAGE PROCESS OF ORDER (NP,NQ)
C
C DIMENSION ALPHA(NPP1),BETA(NQP1),Y(NOBS),AL(IROWS1,IROWS1),
1D(IROWS1),WKL(NQP1),R(NQP1),E(IROWS2),X(NOBS),YPD(NN)
DATA ZERO,IOPTB,IOPTE/0.0,0.1/
C
C TEST FOR INVALID PARAMETERS
C
C
C IFault=1
C IF(NPP1.LT.1) GO TO 160
C IFault=2
C IF(NQP1.LT.1) GO TO 160
C IFault=3
C IF((NVL-NTF+1)*(NVL-NVF+1).GT.NN) GO TO 160
C IFault=4
C IF(IROWS1.LT.NQP1) GO TO 160
C
C FIND E ARRAY
C
C
C NQ=NQP1-1
C NP=NPP1-1
C NPPNQ=NP+NQ
C IF(NQ.EQ.0) GO TO 40
C NX=NROWS-NP
C DO 20 I=1,NX
C NPP1=NP+I
C E=Y(NPP1)
C DO 10 J=1,NP
C NPP1M1=NPP1-J
C E=E+ALPHA(J)*Y(NPP1M1)
10  CONTINUE
C X(I)=E
20  CONTINUE
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C
      CALL MAORTH(IOPTR,IOPTE,NQ,DEL,NX,NX,X,BETA,SIGSQ,IROWS1,
      1R,RO,WKL,D,AL,E,IF1)

C      IFAULT=5
C      IF(IF1.EQ.5) GO TO 160
      DO 30 I=1,NX
      N1=NQBS-I+1
      N2=N1-NP
      E(N1)=E(N2)
  30  CONTINUE

C      FIND PREDICTORS
C
      40  CONTINUE
      N1=NVL-NVF+1
      N2=NP+NVF
      DO 150 NT=NTF,NTL
      N3=(NT-NTF)*N1
      NTEMP1=NT-NP-NQ
      NTEMP=NT-NP
      IF((NPPNQ.EQ.0).OR.(NT.LT.1).OR.(NT.GT.NQBS)) GO TO 55
      IF((NP.NE.0).AND.(NT.LT.NPPNQ)) GO TO 55
      GO TO 60

  55  CONTINUE
      DO 50 I=1,N1
      N3PI=N3+I
      YPD(N3PI)=ZERO
  50  CONTINUE
      GO TO 150

  60  CONTINUE
      IF(NP.EQ.0) GO TO 80
      DO 70 I=1,NP
      N4=NTEMP+I
      X(I)=Y(N4)
  70  CONTINUE

  80  CONTINUE
      DO 130 NV=1,NVL
      C=ZERO
      IF((NV.EQ.0).OR.(NV.GT.NQ)) GO TO 100
      NTPNU=NT+NV
      IF((NP.EQ.0).AND.(NTPNU.LE.NQ)) GO TO 100
      DO 90 K=NV,NQ
      KK=NTPNU-K
      C=C+BETA(K)*E(KK)
  90  CONTINUE
  100  CONTINUE
      NPPNU=NP+NV
      IF(NP.EQ.0) GO TO 120
      DO 110 I=1,NP
      NP1=NPPNU-I
      C=C-ALPHA(I)*X(NP1)
  110  CONTINUE
  120  CONTINUE
      X(NPPNU)=C
  130  CONTINUE
      DO 140 NV=1,N1
      N3PNV=N3+NV
      N5=N2+NV-1
      YPD(N3PNV)=X(N5)
  140  CONTINUE
  150  CONTINUE

C      IFAULT=0
  160  CONTINUE
      RETURN
      END

      SUBROUTINE MAORTH(IOPTR,IOPTE,NQ,DEL,ITERS,IROWS1,Y,BETA,
      1SIGSQ,IROWS2,R,RO,WKL,D,AL,E,IFault)

C      THIS SUBROUTINE CALCULATES THE MODIFIED CHOLESKY FACTORS OF THE
C      ITERS ORDER CORRELATION MATRIX FOR A MOVING AVERAGE PROCESS OF
C      ORDER NQ. IT IS USED TO CALCULATE THE CORRESPONDING COEFFICIENTS
C      AND/OR AN ORTHOGONAL TRANSFORMATION OF INPUTTED MOVING AVERAGE DATA.
  
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      DIMENSION Y(IROWS1),BETA(NQ),R(NQ),WKL(NQ),D(IROWS2),
      1AL(IROWS2),IROWS2),E(IROWS1)
      DATA ONE/1.0/
      DATA EPS/1.E-10/
C      EPS IS CRITERION FOR TESTING FOR ZERO DIAGONAL
C      TEST FOR INVALID PARAMETERS
C
C      IFAULT=1
C      IF(NQ.LT.1) GO TO 299
C      IFAULT=2
C      IF((IOPTB.NE.0).AND.(IOPTB.NE.1)) GO TO 299
C      IFAULT=3
C      IF((IOPTE.NE.0).AND.(IOPTE.NE.1)) GO TO 299
C      IFAULT=4
C      IF(IROWS2.LE.NQ) GO TO 299
C
C      AUTOCORRELATIONS AND INITIALIZATION
C
C      IF(IOPTB.EQ.0) CALL MACV1(NQ,BETA,SIGSQ,R,R0,IF1)
C      D(1)=ONE
C      NQP1=NQ+1
C      DO 5 I=1,NQ
C      R(I)=R(I)/R0
C      5 CONTINUE
C      DO 10 I=1,NQP1
C      AL(I,I)=ONE
C      10 CONTINUE
C      IF(IOPTE.EQ.1) E(1)=Y(1)
C
C      ROW K OF FACTORIZATION
C      M1 IS NUMBER OF LEADING ZEROS
C      M3 IS NUMBER OF ELEMENTS TO CALCULATE
C      MR IS SHIFTING FACTOR IN STORAGE OF PREVIOUS ROWS
C      INDD+J-1 IS ROW NUMBER OF AL FOR CALCULATING WKL(J)

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      DO 130 K=2,ITERS
      M1=MAX0(K-NQ-1,0)
      M3=MING(K-1,NQ)
      INDR=K-M1-1
      MR=MAX0(K-NQ-2,0)
      INDD=M1+1-MR
      WKL(1)=R(INDR)/D(INDD)
      IF(M3.LT.2) GO TO 40
      DO 30 J=2,M3
      JM1=J-1
      INDR1=INDR-JM1
      C=R(INDR1)
      INDLR=INDD+JM1
      M4=-MAX0(M1+J-NQ,1)+M1+1
      DO 20 I=1,JM1
      INDD1=INDD+I-1
      INDL1=M4+I
      C=C-D(INDD1)*WKL(I)*AL(INDL1,INDL1)
      20 CONTINUE
      WKL(J)=C/D(INDLR)
      30 CONTINUE
      40 CONTINUE
      DK=ONE
      DO 50 J=1,M3
      INDD=M1+J-MR
      DK=DK-D(INDD)*WKL(J)*WKL(J)
      50 CONTINUE
      IF(IOPTE.EQ.0) GO TO 65
      C=Y(K)
      DO 60 I=1,M3
      INDWKL=M3-I+1
      INDE=K-I
      C=C-WKL(INDWKL)*E(INE)
      60 CONTINUE
      E(K)=C

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65  CONTINUE
  IFAULT=5
  IF(DK.LT.EPS) GO TO 199
  IF(K.GT.NQF1) GO TO 80
  D(K)=DK
    DO 70 I=1,NQ
      AL(K,I)=WKL(I)
70  CONTINUE
  GO TO 130
80  CONTINUE
  DO 82 I=1,NQ
    IP1=I+1
    D(I)=D(IP1)
    DO 81 J=1,NQ
      AL(I,J)=AL(IP1,J)
81  CONTINUE
82  CONTINUE
  D(NQF1)=DK
  DO 83 I=1,NQ
    AL(NQF1,I)=WKL(I)
83  CONTINUE
C
C  DEL IS CONVERGENCE CRITERION
C
  IF(IOPTB.EQ.0) GO TO 110
  DO 90 I=1,NQ
    IF(ABS(AL(NQF1,I)-AL(NQ,I)).GE.DEL) GO TO 130
90  CONTINUE
  IF(R0*ABS(D(NQF1)-D(NQ)).GE.DEL) GO TO 130
  DO 100 I=1,NQ
    INDWKL=NQ-I+1
    BETA(I)=WKL(INDWKL)
100 CONTINUE
  SIGSQ=R0*D(NQF1)
  IFAULT=0
  GO TO 150

110 CONTINUE
  DO 120 I=1,NQ
    INDB=NQ-I+1
    IF(ABS(WKL(I)-BETA(INDB)).GE.DEL) GO TO 130
120 CONTINUE
  IF(ABS(D(NQF1)*R0-SIGSQ).LT.DEL) GO TO 150
130 CONTINUE
  IFAULT=6
  IF(IOPTB.EQ.0) GO TO 199
  DO 140 I=1,NQ
    INDWKL=NQ-I+1
    BETA(I)=WKL(INDWKL)
140 CONTINUE
  SIGSQ=R0*DK
  GO TO 199
150 CONTINUE
  IFAULT=0
  IF(IOPTE.EQ.0) GO TO 199
  IF(K.EQ.ITER1) GO TO 199
  KP1=K+1
  DO 170 J=KP1,ITER1
    C=Y(J)
    DO 160 I=1,NQ
      INDE=J-I
      C=C-BETA(I)*E(INDE)
160 CONTINUE
    E(J)=C
170 CONTINUE
  IFAULT=0
199 CONTINUE
  DO 200 I=1,NQ
    R(I)=R(I)*R0
200 CONTINUE
299 RETURN
END
SUBROUTINE MACV1(NQ,BETA,SIGSQ,R,R0,IFault)

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C
C THIS SUBROUTINE CALCULATES AUTOCOVARIANCES OF A MOVING AVERAGE
C PROCESS OF ORDER NQ GIVEN ITS COEFFICIENTS AND RESIDUAL VARIANCE.
C
C
      DIMENSION BETA(NQ),R(NQ)
      DATA ONE /1.0/
      IFAULT=1
      IF(NQ.LT.1) GO TO 40
      C=ONE
      DO 10 I=1,NQ
      C=C+BETA(I)*BETA(I)
10    CONTINUE
      R0=SIGSQMC
      DO 20 IV=1,NQ
      NQMIV=NQ-IV
      C=BETA(IV)
      DO 30 I=1,NQMIV
      IVPI=IV+I
      C=C+BETA(I)*BETA(IVPI)
30    CONTINUE
      R(IV)=C*SIGSQ
20    CONTINUE
C
C
      IFAULT=0
40    RETURN
      END
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